1 Ratios and Proportions 2.6

By the end of this section, you should be able to solve the following problems.

1. Write the ratio in lowest term: 8 pints to 5 quarts.
2. Decide if the proportion is true: $\frac{4}{7} = \frac{12}{21}$
3. Find K: $\frac{4}{7} = \frac{24}{K}$
4. A small tree 15 ft high casts a shadow which is 12ft long. How long is the shadow of another three which is 9 ft 7 inches long.

2 Concepts

A ratio is a comparison of two quantities that have the same units. So, if we wish to compare 5 dollars to 6 quarters, we write both quantities as either dollars or quarters. For example, we may write:

$$\frac{5\ dollars}{6\ quarters}$$

$$\frac{20\ quarters}{6\ quarters} = \frac{10}{3}$$

Or we may also write:

$$\frac{5dollars}{1.5dollars} = \frac{5}{1.5}$$

The goal is always to write the ratio in the same units and reduce to lowest terms.

A proportion is an equation between two ratios. In general, we write $\frac{a}{b} = \frac{c}{d}$ for $b \neq 0$ and $d \neq 0$.

A proportion $\frac{a}{b} = \frac{c}{d}$ is true if and only if the products on the diagonals of the equations are equal. That is, $\frac{a}{b} = \frac{c}{d} \Rightarrow a \cdot d = b \cdot c$

We may check the truth value of the statement.

$$\frac{3}{7} = \frac{9}{21}$$

by multiplying diagonally:

$$3 \times 21 = 7 \times 9$$

Since

$$63 = 63,$$

we know that our proportion is a true statement.

By the same token, we may solve any proportional statements that contains an unknown, simply by multiplying diagonally and then dividing to get

result. For example,

$$\frac{3}{4} = \frac{21}{x}$$

$$3x = 84$$

$$\frac{3x}{3} = \frac{84}{3}$$

$$x = 28$$

A proportion can be used to solve problems in the real world. Below is an example.

A certain medicine is to be administered to a baby. If $\frac{1}{2}$ teaspoon is to be administered for every 10 pounds of the baby's weight, how much medicine should be given to a baby that weighs 30 pounds.

We set up our proportion as follows:

$$\frac{Medicine}{Weight} = \frac{Medicine}{Weight}$$

Therefore,

$$\frac{\frac{1}{2}}{10} = \frac{x}{30}$$

$$\frac{1}{2} \times 30 = 10 \times x$$

$$15 = 10x$$

$$\frac{15}{10} = \frac{10x}{10}$$

$$1.5 = x$$

Therefore, the baby must be given 1.5 teaspoons of medicine.

3 Facts

1. A ratio is a quotient of two numbers written as $\frac{a}{b}$ $b \neq 0$ with the restriction that a and b have the same units.

2. A proportion $\frac{a}{b} = \frac{c}{d}$ is true if and only if $ad = bc$, $(b, d \neq 0)$.

4 Exercises

1. Write the ratio in lowest terms: 8 pints to 5 quarts.

2. Decide if the proportion is true:

$$\frac{4}{7} = \frac{12}{21}$$

3. Find K

$$\frac{4}{7} = \frac{24}{K}$$

4. A small tree is 15 ft high and casts a shadow which is 12 ft long. How long is the shadow of another tree 9 ft 7 inches high.

5 Solutions

1.

$$\frac{8\ pints}{5\ quarts} = \frac{8\ pints}{10\ pints}$$

$$\frac{8\ pints}{10\ pints} = \frac{4}{5}$$

2.

$$\frac{4}{7} = \frac{12}{21}$$

$$4 \times 21 = 7 \times 12$$

$$84 = 84$$

The proportion is a true statement.

3.

$$\frac{4}{7} = \frac{24}{K}$$

$$4K = 168$$

$$\frac{4K}{4} = \frac{168}{4}$$

$$K = 42$$

4. We start by setting up our proportion:

$$\frac{Height}{Shadow} = \frac{Height}{Shadow}$$

$$\frac{15ft}{12ft} = \frac{9ft\ 7in}{x}$$

Reducing we have:

$$\frac{5ft}{4} = \frac{9ft\ 7in}{x}$$

Converting to pure inches we have:

$$\frac{60}{36} = \frac{115}{x}$$

$$60x = 4140$$

$$\frac{60x}{60} = \frac{4140}{60}$$

$$x = 69$$

Converting back to inches we have

$$x = 5ft\ 9inches$$